Institut for Matematik og Datalogi
Syddansk Universitet

September 27, 2012
JBJ

DM208 – Fall 2012 – Weekly Note 6

Handout material in week 39:
You will find this in my mailbox in the secretarys office.

- Sections 8.6, 8.7 from Korte and Vygen, Combinatorial Optimization 3rd ed., Springer Verlag.
- Sections 14.5 and 14.6 from D. Jungnickel, Graphs, Networks and Algorithms 3rd edition, Springer Verlag.

Stuff covered in week 39

- Arc-disjoint branchings. This is BJG Section 9.5.
- Minimum cost branchings. This is based on pages 338-341 in the second edition of BJG. These pages are available from the course page.
- Matroid intersection and partition (union). This is based on PS section 12.5, SCH 10.4-10.5 and Korte and Vygen sections 13.5-13.7 (handed out september 23, if you did not get it pick it up in my office).

Lecture October 2, 2012:

- Intersection of 3 or more matroids is NP-complete.
- Gomory-Hu trees. This will be based on hand out material (Korte and Vygen Section 8.6).

Lecture, October 3, 2012:

- Schrijver application 5.5 page 89. This is the Chinese postman problem. See also Jungnickel Section 14.5.
- Shortest paths in undirected graphs with negative edge weights. We will show how to reduce this problem to a minimum weight f-factor problem. Based on Jungnickel Section 14.6.

Problems and exercises for Week 40

- Selected exercises from weekly note 5.
- Let $M_1 = (S, \mathcal{F}_1)$ and $M_2 = (S, \mathcal{F}_2)$ be matroids on the same groundset. Define $\mathcal{F}_{1\cap 2}$ to be
$$\mathcal{F}_{1\cap 2} = \{X \subset S | X \in \mathcal{F}_1 \cap \mathcal{F}_2\}$$
Prove by example that $\mathcal{F}_{1\cap 2}$ need not be the set of independent sets of any matroid.
- Prove that the rank function of the dual matroid M^* of $M = (S, \mathcal{F})$ is given by the equation
$$r^*(X) = |X| + r(S - X) - r(S),$$
where r is the rank function of M.
- This problem concerns matroid intersection and the algorithm described in Korte and Vygen section 13.5. A **multigraph** is a graph in which we allow parallel edges.

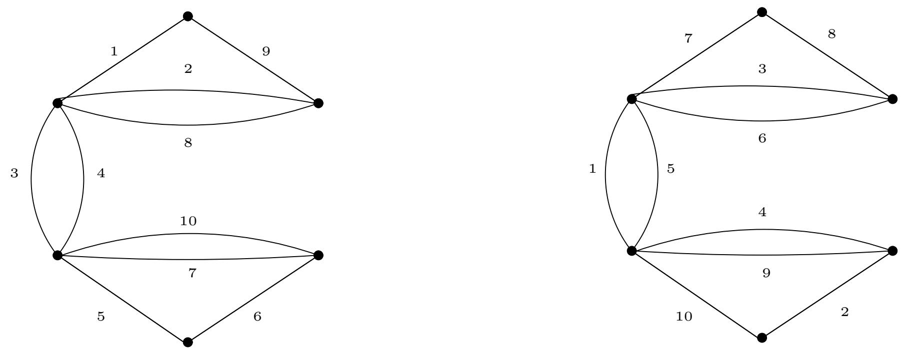

Figure 1: Two multigraphs and the relation between their edges (the numbers illustrate the mappings f_1, f_2). G_1 is the left graph and G_2 the right one.

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two multigraphs with the same number m of edges and let $f_i : E_i \to \{1, 2, \ldots, m\}$, $i = 1, 2$, be bijections. Extend f to subsets of E_i in the obvious way: $f_i(E_i') = \{f_i(e) : e \in E_i'\}$.

Consider the problem of finding a maximum cardinality set E' of edges in G_1 so that E' induces a forrest (contains no cycles) in G_1 and $f_2^{-1}(f_1(E'))$ induces a forrest in G_2. Equivalently: we are looking for a maximum cardinality subset S of $\{1, 2, \ldots, m\}$ so that the edges with numbers from S form a forrest in G_i for $i = 1, 2$.

Question a:

Show how to formulate this problem as a matroid intersection problem. Hint: you may use $\{1, 2, \ldots, m\}$ as the ground set for both matroids.

Question b:

Let G_1, G_2 be the graphs in Figure 1 where the number on the edges in G_i gives the functions f_i, $i = 1, 2$, i.e. the numbering of the edges. Argue that with these numberings the images (under f_i^{-1}, $i = 1, 2$) of the set $X = \{1, 2, 3, 10\}$ is a forrest in both graphs but no further element can be added to X while preserving this property in both graphs.

Question c:

Draw the graph G_X (described in Section 13.5 in Korte and Vygen) and describe how you obtain its arc set.

Question d:

Describe a directed path in G_X which shows that X is not a maximal common independent set of your matroids and give the resulting common spanning tree in both graphs.