Institut for Matematik & Datalogi
Syddansk Universitet

December 17, 2010
JBJ

DM528: Combinatorics, Probability and Randomized Algorithms — Ugeseddel 7

Lecture Monday December 20, 2010:

- Kleinberg and Tardos Section 13.11
- Overview of the course.
- Hints for the exam.

Exercises Monday December 20 and Wednesday December 22 , 2010:

- Consider an experiment in which we flip a coin $2n$ times and let X_h denote the number of heads we obtain and let X_t denote the number of tails.
  - Find the expected values of X_h, X_t.
  - Find the variance of X_h, X_t. Hint use Example 18, Rosen page 438.
  - Prove that for every $\epsilon > 0$ there exists a constant $c > 0$ such that

    $p[X_h - X_t > c\sqrt{n}] \leq \epsilon$. Hint: note that $X_h - X_t = 2(X_h - n)$ and use Chebyshev's inequality.
  - Suppose $n = 100$. Find a bound for the probability that $X_h - X_t \geq 20$. Then do the same for $n = 1000$ and probability that $X_h - X_t \geq 200$. Hint: use Chernoff bounds.
- Randomized median approximation (rewriting of problem from Kleinberg and Tardós): Suppose you are faced with finding the median of a very large set S of real numbers and you would like to approximate the median of these numbers by sampling (that is, selecting some subset of the elements and working only on that). Assume that all numbers in S are distinct. Let $n = |S|$ and call x an **ϵ-approximate** median of S if at least $(\frac{1}{2} - \epsilon)n$ numbers of S are smaller than x and at least $(\frac{1}{2} - \epsilon)n$ numbers of S are larger than x. Consider the following algorithm: first select a subset $S' \subseteq S$ uniformly at random, then compute the median m of S' and return this m as the ϵ-approximate median for S. Show that there is an absolute constant c independent of n, so that if you apply this algorithm with a sample S' of size c, then with probability at least 0.99, the number m returned will be a (0.05)-approximate median of S (you may allow sampling with replacement if you wish). Hint: what is the probability that a randomly chosen element from S is an ϵ-approximate median for S?
- Rosen Section 7.4: 10(a),(b),(d), 12(a),(b), 14, 16, 22, 24, 26(a),32,34, 50, 52.

- Previous exam problems:
  - 2004.06.6 page 5 if you did not do that earlier.
  - 2004.08.5 page 7.
  - 2005.13.4 page 16.
- In case of more time you should talk about the assignments for homework 2.