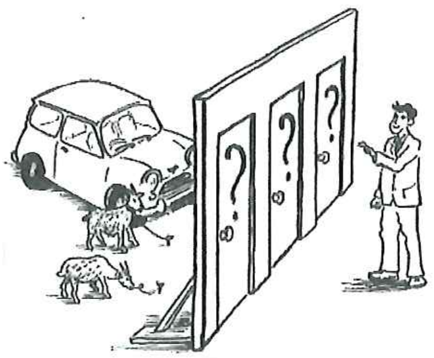
Readers' Challenge report: The Monty Hall problem

In August we invited readers to explain the Monty Hall problem briefly and clearly. The challenge attracted a record postbag.



It all began with a TV quiz show of the 1960s. The show itself is long forgotten. The questions of probability that it raised have remained and are paradoxical, contentious and confusing to this day.

The contestant chooses one of three doors. Behind one door is a car and behind the other two are goats. The host, Monty Hall, (who knows where the car is) opens one of the remaining doors to reveal a goat, and invites the contestant to 'stick' with his original choice of door – or 'switch' to the other closed door. Does sticking or switching give the best chance of winning the car?

It seems simple, but to many the true answer is counter-intuitive: the great mathematician Paul Erdös refused to believe it for years. As Mark Robson said in his reply, funny how muddled even brilliant minds can become in the face of conditionality. And when Peter Calver published his (correct) solution in a local magazine some of the responses from readers were 'I'm sorry but where did you learn about probability?'; 'I am surprised that you have not consulted someone with an elementary understanding of probability theory'; 'I am a professional who works with probability for a living. I'm afraid it is you who are wrong'; and 'Take my advice and don't try gambling, I don't think you are cut out for it.' ... So clearly Monty Hall is a big nut to crack.

It attracted a correspondingly big postbag – the largest we have yet had. Answers could be divided roughly into the mathematical and the verbal.

A mathematical solution came from Trevor Sharot in Singapore:

> If the contestant initially chooses correctly, which has $p = 1/3$, switching is a losing strategy. If the contestant initially chooses incorrectly, which has $p = 2/3$, switching is a winning strategy. Thus overall, switching wins with $p = 2/3$.
>
> We compare this to the probability of winning if the contestant does not switch.
>
> If the contestant initially chooses correctly, which has $p = 1/3$, not switching is a winning strategy. If the contestant initially chooses incorrectly, which has $p = 2/3$, not switching is a losing strategy. Thus overall, not switching wins with $p = 1/3$.
>
> Switching therefore wins twice as often as not switching.

Which is good and clear and simple for the mathematically inclined, but might well make other heads swim.

Lawrence Lesser of El Paso in Texas gave it diagrammatically:

> Write out the sample space (see table below), and mentally 'pick a door' (say Door 1.) Now, one scenario-row at a time, mentally 'open another door' (as Monty does) that is neither your door nor where the car is, and then ask yourself 'am I better off sticking with my original choice?' In two of the three rows, you will be better off – you will win the car – by switching from your original choice to the other unopened door. Thus, the probability of winning by switching is 2/3.

Scenario	*Probability*	*Door1*	*Door2*	*Door3*
1	1/3	Car	Goat	Goat
2	1/3	Goat	Car	Goat
3	1/3	Goat	Goat	Car

The *reductio ad absurdam* argument was also popular. It relies on increasing the number of doors. If there were 100 doors, and 99 goats, and the contestant chooses one door and Monty Hall opens all the other doors except number 72, and all of them have goats, the car must be behind the contestant's door or the suspiciously still-unopened number 72. In this thought experiment it is intuitively obvious that the car is behind door 72. As David Kerr put it, 'Once you go through this thought experiment, the value of switching doors in the three door situation becomes clear as well.'

Mark Robson gave the same argument by analogy with playing cards:

> I lay out 52 cards face down. I know exactly where I have put the Ace of Spades – and I tell you that I know, before inviting you to guess which one it is.
>
> You make a truly random guess. The probability that you are correct is 1/52.
>
> I then turn over 50 cards, leaving just the one you have selected and one other. You can see that none of those I have turned over is the Ace of Spades. Only two remain, of which one is that card. I invite you to maintain your original choice, or to switch to the other card.
>
> Quite obviously (I contend) you should switch in this case. For the probability that the one you did not choose is in fact the Ace of Spades is clearly not 1/2 but 51/52. I have almost surely shown you by my disclosures which card is the Ace of Spades – except in the unlikely event that in fact you originally guessed correctly. Most people can understand this, either in the abstract or with a demonstration.

Do it again but this time with just a three card pack – the Ace of Spades and two 'goat' cards – and you can reproduce all the elements of the Monty Hall problem in the safety and comfort of your home.

The prize for economy of words goes to Gib Bassett:

> You choose door A.
>
> (i) You stay with door A.
> **Result:** you win *only if* the car is behind A – 1/3 of the time.
>
> (ii) You switch to the 'other' door.
> **Result:** you lose *only if* the car is behind A – 1/3 of the time. (Hence you win 2/3 of the time.) Monty *never*

opens a door with the car. So, if the car is not behind A it will be behind the 'other' door.

As many of you pointed out, the key insight is that no matter which door you select, Monty can always show you a goat behind one of the remaining doors. His action provides no new information about your selection but *does* provide information about the other two doors. 'What throws people off in this problem is the tendency to assume that the two unopened doors – yours and Monty's – are equally likely to hide the car, when in fact this assumption is not at all obvious and, actually, it's wrong.' said Daniel Gervini. Stephen Springate points out that the answer changes if Monty Hall does not know which door conceals the car:

> As a final paradoxical twist, had the host *not* known which door concealed the prize but had by chance opened exactly the same door with no prize behind it, the two remaining doors would then have had an equal (1 in 2) chance of concealing the prize.

So, changing your choice in this variant would neither increase nor decrease your chance of winning the car.

The verbal explanations, on the whole, made it clearer to the editor. Peter Button helped by labelling the goats:

> Behind one door is a car and behind the other two are goats A and B respectively. The probability of initially selecting the door hiding the car is 1/3, the probability of selecting the door hiding goat A is 1/3 and the probability of selecting the door hiding goat B is 1/3.
>
> Monty Hall then opens a door revealing either goat A or goat B – so the two remaining closed doors are hiding a car and a goat.
>
> If the contestant had initially selected the door hiding the car and they 'stick' they would win, if they 'switch' they lose. If they had selected the door hiding goat A and then 'stick' they lose, and if they 'switch' they win. If they had selected the door hiding goat B and then 'stick' they lose, and if they 'switch' they win.
>
> So, if the contestant chooses to 'stick' they would win one out of three times, or 33%, if they choose to 'switch' they would win two out of three times, or 67%. So the best strategy would be to 'switch'.

My favourite, for simplicity, clarity, and making you intuitively understand the answer came from Susan Gruber:

> We know that behind two of the doors there is a goat, and behind the third door is a car. Imagine that you select a door at random. What is behind it? Two out of three times, it will be a goat. Now that you have chosen a goat, what is behind the remaining doors? Monty Hall knows what is behind each one, and opens a door to reveal the second goat. What is behind the third, unopened door? It must be the car. Two out of three times, this is the way the scenario plays out. You pick a goat, Monty shows you a second goat, and the car is behind the remaining, unopened door. Of course, one out of three times it plays out differently. One out of three times, you pick the car. Then Monty opens a door to reveal a goat, and behind the remaining door is the second goat. One third of the time, when you swap your door for the unopened door, you are swapping a car for a goat. But, as we've seen, two thirds of the time you will be swapping a goat for a car.

So it is better to switch. Sorted.

Julian Champkin

Statistical Battleships

Our June challenge was the schoolboy game of Battleships, in which contestants aim at squares on a ten by ten grid in the hope of 'hitting' their opponent's ships: an Aircraft Carrier (5 squares), a Battleship (4 squares), a Destroyer and a Submarine (each 3 squares) and a Patrol Boat (2 squares.) (See *Significance* June 2013 for the full rules.) Which aiming strategy gives most chance of winning?

A small but select postbag produced opposing ideas. Christine and John Blundell, of Terra Alta, West Virginia, are clearly keen players:

> The smallest ship, and the hardest to find, is the Patrol Boat (two squares). Given a total of 100 squares, a firing pattern of 50 shells into alternate squares that touch at corners but nowhere else must by definition discover this tiny ship and incidentally all the others. And the larger two ships by definition must by then have several hits.
>
> So first find that little boat. You cannot win without sinking it. The moment you have done that your firing pattern changes to every third square, and so on. Hitting all odd numbers on A, C, E, G and I and all even numbers on B, D, F, H and J or vice versa leaves no place to hide.

Curiously, they recommend not finishing off a ship once hit:

> Once you hit a ship do not try to sink it. Just mark the hit and collect the information from your enemy as to what size it is (2, 3, 4 or 5 squares long); keep the shelling pattern going (maybe changed) as this will reveal intelligence as to whether the ship is vertical or horizontal thus saving shots later.

A snag with that is that in the standard rules you do not learn what size ship you have hit until you have sunk it.

But Ian Blois of Inverness pointed out that the 'ships' are large compared to the 'sea'. This means that not all squares on the grid are equally likely to hold ships – some ships must stick out away from the corners. The Aircraft Carrier is 5 squares long – half the width of the sea; at least part of it must therefore occupy squares in columns 5 or 6 of the grid, if the carrier is sailing horizontally, or in rows E or F if it is vertical. So at least one of these 36 squares must be occupied – giving a probability of at least 1 in 36 = .0277 of scoring a hit if you aim at those squares. Altogether there are 5+4+3+3+2 = 17 occupied squares in the 10 × 10 grid, giving 17/100 = .017 chance of a hit if you aim randomly. So aiming at the central rows and columns gives a better score rate; start there and work outwards. And once you have hit the aircraft carrier, go on to destroy it. That will eliminate the squares all around it, since ships cannot touch.

Probably combining the two methods will work best. Readers may care to report empirical results.

This month's challenge is on page 5.