Institut for Matematik og Datalogi
Syddansk Universitet

November 17, 2022
JBJ

DM817 – Fall 2022 – Weekly Note 8.

Stuff covered in week 46, 2022

- BJG Section 7.3, proof of Menger's theorem via submodularity [Video 23]
- BJG 7.5-7.6 on Mader's splitting theorem and arc-connectivity augmentation [Video 24]

Video Lectures in Week 47

- Convex cost flows [Video 25]. Based on Ahuja Chapter 14.
- Arc-disjoint branchings [Video 26]. Based on BJG Section 9.5

New exercises

- 7.28 and 7.30
- Ahuja 14.4, 14.5, 14.14, 14.15 and 14.17
- In this exercise graphs and digraphs may have parallel edges and arcs.

  A digraph $D = (V, A)$ is **eulerian** if $d^-(x) = d^+(x)$ for every vertex $x \in V$[1]. An **eulerian orientation** of an undirected graph $G = (V, E)$ is an eulerian digraph D that is obtained from G by assigning an orientation to each edge of G. It is a well known fact that a connected undirected graph G has an eulerian orientation if and only if the degree of every vertex in G is even.

  Suppose now that we are given a digraph $D = (V, A)$ which is not eulerian but satisfies that $d^-(x) + d^+(x)$ is even for all $x \in V$. Then the result above implies that it is possible to reorient a subset X of the arcs of D so that we obtain a new digraph $D' = (V, A')$ which is eulerian.

[1] NB: This does not mean that D must be regular, as some vertex x may have in- and out-degree p and another vertex y in- and out-degree q with $q \neq p$.

Question a:

Explain how to formulate the problem of finding a such set X of arcs in D as a feasible flow problem. Hint: use the flow variable to indicate whether an arc is reversed or not and express the resulting in-degree of a vertex v in terms of the in-degree in D and the values of x on arcs incident with v.

Question b:

Explain how we can use the flow model above to develop a polynomial algorithm for finding the minimum number of arcs we need to reverse in D in order to obtain an eulerian digraph D'.

Question c:

Explain how to use the flow model above to develop a polynomial algorithm for finding the maximum number of arcs we can reverse in D in order to obtain an eulerian digraph D''.

Suppose now that we would like to minimize the maximum number of arcs that are reversed at any vertex, that is, we wish to determine the minimum k such that D can be made eulerian by reversing at most k arcs at any vertex $v \in V(D)$.

Question d:

Show how to formulate this problem as a convex cost flow problem. Hint: what is the absolute minimum number of arcs we must reverse at a given vertex and how do we avoid reversing more extra arcs than necessary at v (and the other vertices of D)?