Packing I'' using dyn. prg.

We will use the same approach as in Section 3.2.

Since all items in I'' have size at least $\varepsilon/2$, at most $2/\varepsilon$ items fit into each bin.

There are at most $N = \lceil n/k \rceil$ different item sizes $s_1, s_2, \dots, s_N$ in I''

Hence, any packing of a bin can be represented by a vector $(m_1, m_2, \dots, m_N)$, $m_i \le 2/\varepsilon$, where m_i is the number of items of size s_i in the bin.
A vector representing the contents of a bin is called a bin configuration.
Let $\mathcal{C}$ be the set of possible bin configurations.
Note that $|\mathcal{C}| \le (2/\varepsilon)^N$.

For the dyn. prg. we will use an N-dimensional table B with n_i+1 rows in the i'th dimension, where n_i is the number of items of size s_i in I''.
$B[m_1, m_2, \dots, m_N]$ will be the minimum number of bins required to pack m_i items of size s_i, $1 \le i \le N$.

Ex:

$\varepsilon = 0.4$

$I = 0.6,\ 0.5,\ 0.4,\ 0.4,\ 0.3,\ \underbrace{0.1,\ 0.1}_{< \varepsilon/2}$

Choosing $k=3$, we obtain

$I' = \underbrace{0.6,\ 0.5,\ 0.4}, \underbrace{0.4,\ 0.3}$

$I'' = 0.6, 0.6, 0.6, 0.4, 0.4$

$s_1 = 0.6, \quad s_2 = 0.4$

$n_1 = 3, \quad n_2 = 2$

$\mathcal{C} = \{(0,1), (0,2), (1,0), (1,1)\}$

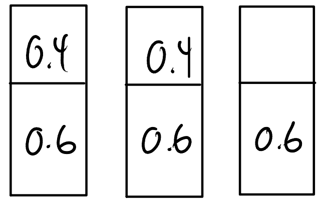
B:

0.6 \ 0.4	0	1	2
0	0	1	1
1	1	1	2
2	1	2	2
3	2	2	3

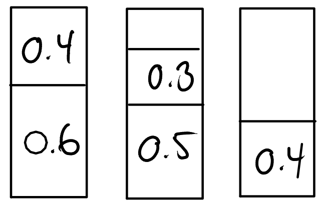
$$B[3,2] = 1 + \min_{(m_1,m_2)\in\mathcal{C}} \{B[3-m_1,\ 2-m_2]\}$$

$$= 1 + \min\{B[3,1],\ B[3,0],\ B[2,2],\ B[2,1]\}$$

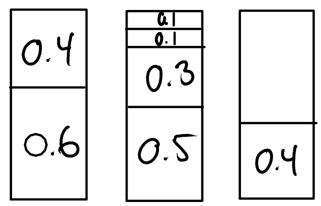
Packing of I'':



Packing of I':



Packing of I:



Running time

$k = \lfloor \varepsilon \cdot size(I) \rfloor \geq \lfloor \varepsilon \cdot n' \cdot \frac{\varepsilon}{2} \rfloor \geq n' \cdot \frac{\varepsilon^2}{4}$, where $n' = |I'|$,

since all items in I' have size at least $\varepsilon/2$.

$N \leq \lceil \frac{n'}{k} \rceil \leq \lceil \frac{4}{\varepsilon^2} \rceil$

Table size $\leq (n')^N \leq n^N$

Time per entry $O(|\mathcal{B}|) \subseteq O\left(\left(\frac{2}{\varepsilon}\right)^N\right)$

Running time $O\left(\left(\frac{2}{\varepsilon}\right)^N n^N\right) \subseteq O\left(\left(\frac{2n}{\varepsilon}\right)^{\lceil 4/\varepsilon^2 \rceil}\right)$

not fully poly. time

Hence, $\{A_\varepsilon\}$ is an

Asymptotic poly. time approx. scheme (APTAS)

This proves:

$\boxed{\text{Theorem 3.12: There is an APTAS for Bin Packing}}$