DM865 – Spring 2020
Heuristics and Approximation Algorithms

Resource Constrained Project Scheduling

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Outline

1. Resource Constrained Project Scheduling Model

2. Preprocessing

3. Heuristics

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RCPSP

Resource Constrained Project Scheduling Model

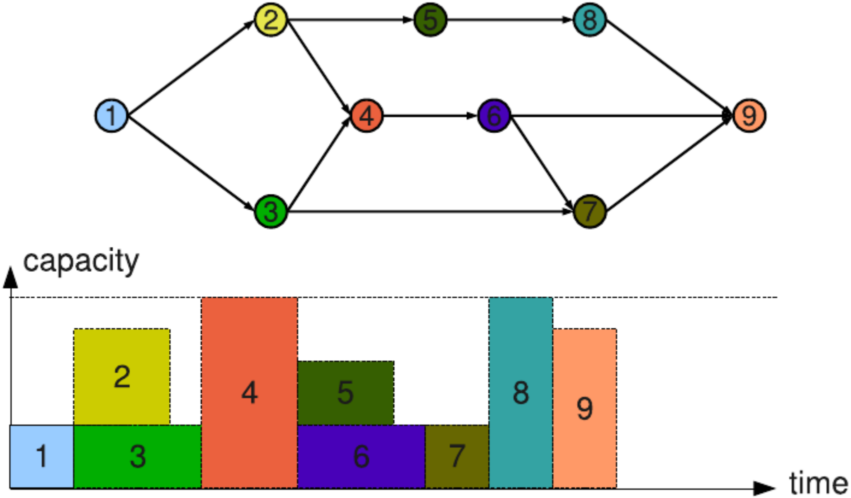
Given:

- activities (jobs) $j = 1, \ldots, n$
- renewable resources $i = 1, \ldots, m$
- amount of resources available R_i
- processing times p_j
- amount of resource used r_{ij}
- precedence constraints $j \rightarrow k$

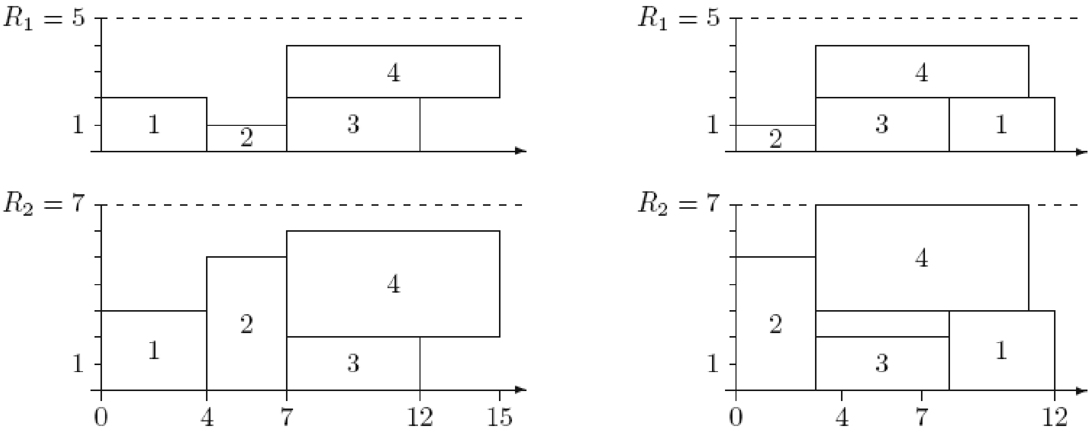
Further generalizations

- Time dependent resource profile $R_i(t)$
  given by (t_i^μ, R_i^μ) where $0 = t_i^1 < t_i^2 < \ldots < t_i^{m_i} = T$
  Disjunctive resource, if $R_k(t) = \{0, 1\}$; cumulative resource, otherwise
- Multiple modes for an activity j
  processing time and use of resource depends on its mode m: p_{jm}, r_{jkm}.

An Example



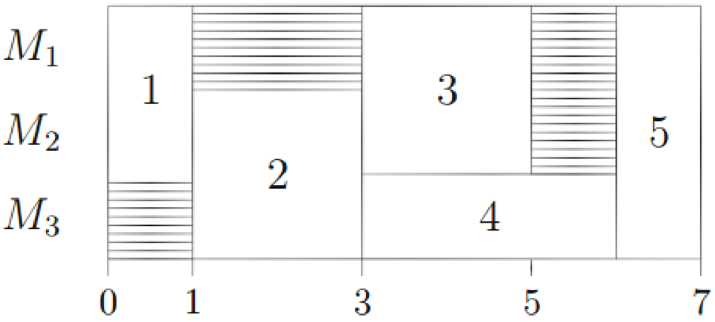
An Example



(a) A feasible schedule (b) An optimal schedule

Multi-processor Task Scheduling

j	1	2	3	4	5
μ_j	$\{M_1, M_2\}$	$\{M_2, M_3\}$	$\{M_1, M_2\}$	$\{M_3\}$	$\{M_1, M_2, M_3\}$
p_j	1	2	2	3	1



Equivalent to a RCPSP with $r = m$ and $R_k = 1$ for $k = 1..m$

Modeling

Assignment 1

- A contractor has to complete n activities.
- The duration of activity j is p_j
- each activity requires a crew of size W_j.
- The activities are not subject to precedence constraints.
- The contractor has W workers at his disposal
- his objective is to complete all n activities in minimum time.

Assignment 2

- Exams in a college may have different duration.
- The exams have to be held in a gym with W seats.
- The enrollment in course j is W_j and
- all W_j students have to take the exam at the same time.
- The goal is to develop a timetable that schedules all n exams in minimum time.
- Consider both the cases in which each student has to attend a single exam as well as the situation in which a student can attend more than one exam.

Assignment 3

- In a basic high-school timetabling problem we are given m classes $c_1, \dots, c_m$,
- h teachers $a_1, \dots, a_h$ and
- T teaching periods $t_1, \dots, t_T$.
- Furthermore, we have lectures $i = l_1, \dots, l_n$.
- Associated with each lecture is a unique teacher and a unique class.
- A teacher a_j may be available only in certain teaching periods.
- The corresponding timetabling problem is to assign the lectures to the teaching periods such that
  - each class has at most one lecture in any time period
  - each teacher has at most one lecture in any time period,
  - each teacher has only to teach in time periods where he is available.

Assignment 4

- A set of jobs $J_1, \ldots, J_g$ are to be processed by auditors $A_1, \ldots, A_m$.
- Job J_l consists of n_l tasks ($l = 1, \ldots, g$).
- There are precedence constraints $i_1 \rightarrow i_2$ between tasks i_1, i_2 of the same job.
- Each job J_l has a release time r_l, a due date d_l and a weight w_l.
- Each task must be processed by exactly one auditor. If task i is processed by auditor A_k, then its processing time is p_{ik}.
- Auditor A_k is available during disjoint time intervals $[s_k^\nu, l_k^\nu]$ ($\nu = 1, \ldots, m$) with $l_k^\nu < s_k^\nu$ for $\nu = 1, \ldots, m_k - 1$.
- Furthermore, the total working time of A_k is bounded from below by H_k^- and from above by H_k^+ with $H_k^- \leq H_k^+$ ($k = 1, \ldots, m$).
- We have to find an assignment $\alpha(i)$ for each task $i = 1, \ldots, n := \sum_{l=1}^{g} n_l$ to an auditor $A_{\alpha(i)}$ such that
  - each task is processed without preemption in a time window of the assigned auditor
  - the total workload of A_k is bounded by H_k^- and H_k^k for $k = 1, \ldots, m$.
  - the precedence constraints are satisfied,
  - all tasks of J_l do not start before time r_l, and
  - the total weighted tardiness $\sum_{l=1}^{g} w_l T_l$ is minimized.

Mathematical Model

$$
\begin{aligned}
\min \quad & \max_{j=1}^{n} \{S_j + p_j\} \\
\text{s.t.} \quad & S_j \geq S_i + p_i, \qquad j = 1, \ldots, n, \forall (i, j) \in A \\
& \sum_{j \in J(t)} r_{jk} \leq R_k, \qquad k = 1, \ldots m, t = 1 \ldots, T \\
& \qquad J(t) = \{j = 1, \ldots, n \mid S_j \leq t \leq S_j + p_j\} \\
& S_j \geq 0, \qquad j = 1, \ldots, n
\end{aligned}
$$

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Preprocessing: Temporal Analysis

- Precedence network must be acyclic

Preprocessing: constraint propagation

1. conjunctions $i \rightarrow j$ $S_i + p_i \leq S_j$
   [precedence constrains]

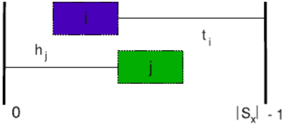
2. parallelity constraints $i \,||\, j$ $S_i + p_i \geq S_j$ and $S_j + p_j \geq S_i$
   [time windows $[r_j, d_j]$,$[r_l, d_l]$ and $p_l + p_j > \max\{d_l, d_j\} - \min\{r_l, r_j\}$]

3. disjunctions $i - j$ $S_i + p_i \leq S_j$ or $S_j + p_j \leq S_i$
   [resource constraints: $r_{jk} + r_{lk} > R_k$]

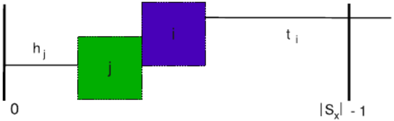
N. Strengthenings: symmetric triples, etc.

Let i, j be a pair of activities. A precedence relation is added between i and j if one of the following holds:

- $h_j + t_i \geq |S_x| - 1$



- $h_j + p_j + p_i + t_i > |S_x| - 1 \quad \wedge \quad \exists k = 1, \ldots, m : r_{ik} + r_{jk} > R_k$



Solutions

Task: Find a schedule indicating the starting time of each activity

- All solution methods restrict the search to feasible schedules, S, S'
- Types of schedules
  - Local left shift (LLS): $S \to S'$ with $S'_j < S_j$ and $S'_l = S_l$ for all $l \neq j$.
  - Global left shift (GLS): LLS passing through infeasible schedule
  - Semi active schedule: no LLS possible
  - Active schedule: no GLS possible
  - Non-delay schedule: no GLS and LLS possible even with preemption
- If regular objectives $\Longrightarrow$ exists an optimum which is active

Hence:

- Schedule not given by start times S_i
  - space too large $O(T^n)$
  - difficult to check feasibility
- Sequence (list, permutation) of activities $\pi = (j_1, \ldots, j_n)$
- π determines the order of activities to be passed to a schedule generation scheme

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Schedule Generation Schemes

Given a sequence of activity, SGS determine the starting times of each activity

Serial schedule generation scheme (SSGS)

n stages, S_λ scheduled jobs, E_λ eligible jobs

Step 1 Select next from E_λ and schedule at earliest.

Step 2 Update E_λ and $R_k(\tau)$.
If E_λ is empty then STOP,
else go to Step 1.

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Procedure Serial Schedule Generation Scheme
 1. Let E_1 be the set of all activities without predecessor;
 2. FOR λ := 1 TO n DO
 3.    Choose an activity j ∈ E_λ;
 4.    t := max_{i→j∈A} {S_i + p_i};
 5.    WHILE a resource k with r_jk > R_k(τ) for some time
       τ ∈ {t+1,...,t+p_j} exists DO
 6.       Calculate the smallest time t_k^μ > t such that j
          can be scheduled in the interval [t_k^μ, t_k^μ + p_j[ if
          only resource k is considered and set t := t_k^μ;
 7.    ENDWHILE
 8.    Schedule j in the interval [S_j, C_j[ := [t, t+p_j[;
 9.    Update the current resource profiles by setting
       R_k(τ) := R_k(τ) − r_jk for k = 1,...,r; τ ∈ {t+1,...,t+p_j};
10.    Let E_{λ+1} := E_λ \ {j} and add to E_{λ+1} all successors
       i ∉ E_λ of j for which all predecessors are scheduled;
11. ENDFOR
```

Parallel schedule generation scheme (PSGS) (Time sweep)

stage λ at time t_λ

S_λ (finished activities), A_λ (activities not yet finished),
E_λ (eligible activities)

Step 1 In each stage select maximal resource-feasible subset of eligible activities in E_λ and schedule it at t_λ.

Step 2 Update E_λ, A_λ and $R_k(\tau)$.
If E_λ is empty then STOP,

else move to $t_{\lambda+1} = \min \left\{ \min_{j \in A_\lambda} C_j, \min_{\substack{k=1,\dots,r \\ i \in m_k}} t_i^\mu \right\}$

and go to Step 1.

- If constant resource, it generates non-delay schedules
- Search space of PSGS is smaller than SSGS

Procedure Parallel Schedule Generation Scheme

1. $\lambda := 1;\ t_1 := 0;\ A_1 := \emptyset$;
2. Let E_1 be the set of all activities i without predecessor and $r_{ik} \leq R_k(\tau)$ for $k = 1, \dots, r$ and all $\tau \in \{1, \dots, p_i\}$;
3. WHILE not all activities are scheduled DO
4.   WHILE $E_\lambda \neq \emptyset$ DO
5.   Choose an activity $j \in E_\lambda$;
6.   Schedule j in the interval $[S_j, C_j[\ := [t_\lambda, t_\lambda + p_j[$;
7.   Update the current resource profiles by setting $R_k(\tau) := R_k(\tau) - r_{jk}$ for $k = 1, \dots, r;\ \tau \in \{t_\lambda + 1, \dots, t_\lambda + p_j\}$;
8.   Add j to A_λ and update the set E_λ by eliminating j and all activities $i \in E_\lambda$ with $r_{ik} > R_k(\tau)$ for some resource k and a time $\tau \in \{t_\lambda + 1, t_\lambda + p_i\}$;
9.   ENDWHILE
10.   Let $t_{\lambda+1}$ be the minimum of the smallest value $t_k^\mu > t_\lambda$ and $\min\limits_{i \in A_\lambda} \{S_i + p_i\}$;
11. $\lambda := \lambda + 1$;
12.   Calculate the new sets A_λ and E_λ;
13. ENDWHILE

Possible uses:

- Forward
- Backward
- Bidirectional
- Forward-backward improvement (justification techniques)
  [V. Valls, F. Ballestín and S. Quintanilla. Justification and RCPSP: A technique that pays. EJOR, 165:375-386, 2005]

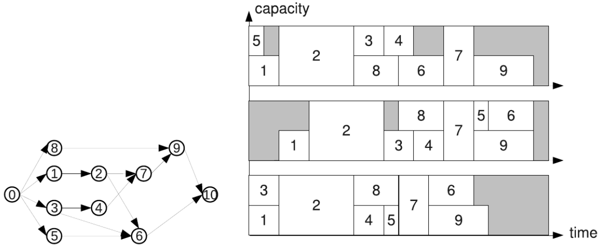

Fig. from [D. Debels, R. Leus, and M. Vanhoucke. A hybrid scatter search/electromagnetism meta-heuristic for project scheduling. EJOR, 169(2):638Â653, 2006]

Dispatching Rules

Determines the sequence of activities to pass to the schedule generation scheme

- activity based
- network based
- path based
- resource based

Static vs Dynamic

Local Search

All typical neighborhood operators can be used:

- Swap
- Interchange
- Insert

reduced to only those moves compatible with precedence constraints

Genetic Algorithms

Recombination operator:

- One point crossover
- Two point crossover
- Uniform crossover

Implementations compatible with precedence constraints

Ant Colony

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Ant algorithm RCPSP
 1.  REPEAT
 2.     FOR k := 1 TO m DO
 3.         FOR i := 1 TO n DO
 4.             Choose an unscheduled eligible activity
                j ∈ V for position i with probability
                p_ij^k = [τ_ij]^α [η_ij]^β / Σ_{l∈V^k} [τ_il]^α [η_il]^β ;
 5.         ENDFOR
 6.     ENDFOR
 7.     Calculate the makespans C^k of the schedules
        constructed by the ants k = 1, ..., m;
 8.     Determine the best makespan C* = min_{k=1}^{m} {C^k} and a
        corresponding list L*;
 9.     FOR ALL activities j ∈ V and their corresponding
        positions i in L* DO
10.         τ_ij := (1 − ϱ)τ_ij + ϱ 1/(2C*);
11.  UNTIL a stopping condition is satisfied
```

$$p_{ij}^k = \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta}{\sum_{l \in V^k} [\tau_{il}]^\alpha [\eta_{il}]^\beta}$$

$$C^* = \min_{k=1}^{m}\{C^k\}$$

$$\tau_{ij} := (1-\varrho)\tau_{ij} + \varrho \frac{1}{2C^*}$$